Exercise

ch 1

Exercise (1)

هندسة إعدادي

1. Find the distance bet$^{\underline{n}}$ the following Points:

P4

(i) $\overrightarrow{(5,1)}^{A}$ and $\overrightarrow{(6,0)}^{B}$

(ii) $\overrightarrow{(2,8)}^{C}$ // $\overrightarrow{(2,-3)}^{D}$

(iii) $\overrightarrow{(1,2)}^{E}$ and $\overrightarrow{(3,4)}^{F}$

Ans: for two Points $P=(x_1,y_1)$. $Q=(x_2,y_2)$. the distance bet$^{\underline{n}}$ them is:

$$PQ=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

(i) $AB=\sqrt{(5-6)^2+(1-0)^2} \Rightarrow \boxed{AB=\sqrt{2}}$ ※

(ii) $CD=\sqrt{(2-2)^2+(8+3)^2} \Rightarrow \boxed{CD=11}$ ※

(iii) $EF=\sqrt{(1-3)^2+(2-4)^2} \Rightarrow \boxed{EF=2\sqrt{2}}$ ※

2. Find the lengthes of the sides of the triangle whose vertices are $(5,1)$, $(-3,7)$ and $(8,5)$ and Prove that one of the angles is a right angle.

P4

Ans: let $A=(5,1)$, $B=(-3,7)$ and $C=(8,5)$

$AB=\sqrt{(5+3)^2+(1-7)^2} \Rightarrow \boxed{AB=10} \Rightarrow (AB)^2=100$

$BC=\sqrt{(-3-8)^2+(7-5)^2} \Rightarrow \boxed{BC=5\sqrt{5}} \Rightarrow (BC)^2=125$

$CA=\sqrt{(5-8)^2+(1-5)^2} \Rightarrow \boxed{CA=5} \Rightarrow (CA)^2=25$

$\because (AB)^2+(CA)^2=125$, $(BC)^2=125$

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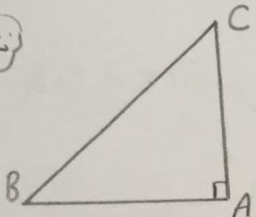

$\therefore (BC)^2=(AB)^2+(CA)^2 \Rightarrow$ "Pythagorean theory"

Then

The Triangle BAC has a right angle at $A=(5,1)$

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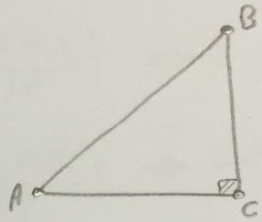
3. Prove that the triangle whose vertices are $A=(-2,1)$, $B=(-1,4)$, $C=(0,3)$ is a right angled triangle.

Ans:

$AB=\sqrt{(-2+1)^2+(1-4)^2} \Rightarrow AB=\sqrt{10}$, $(AB)^2=10$

, $BC=\sqrt{(-1-0)^2+(4-3)^2} \Rightarrow BC=\sqrt{2}$, $(BC)^2=2$

, $CA=\sqrt{(-2-0)^2+(1-3)^2} \Rightarrow CA=2\sqrt{2}$, $(CA)^2=8$



$\because (CA)^2+(BC)^2=10$, $(AB)^2=10$, i.e.

$(AB)^2=(CA)^2+(BC)^2 \longrightarrow$ Pythagorean theory.

$\therefore$ The triangle ABC is right angled at C

4. Show that the points $\underset{A}{(a,a)}$, $\underset{B}{(-a,-a)}$ and $\underset{C}{(-a\sqrt{3},\ a\sqrt{3})}$ are vertices of equilateral triangle.

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Ans: $AB=\sqrt{(a+a)^2+(a+a)^2} \Rightarrow AB=2\sqrt{2}\,a$

$BC=\sqrt{(-a+a\sqrt{3})^2+(-a-a\sqrt{3})^2}=2\sqrt{2}\,a$

$CA=\sqrt{(a+a\sqrt{3})^2+(a-a\sqrt{3})^2}=2\sqrt{2}\,a$

$\because AB=BC=CA \Rightarrow ABC$ is equilateral triangle.

P4

5. Show that the four points $\underset{A}{(1,0)}$, $\underset{B}{(6,1)}$, $\underset{C}{(5,6)}$ and $\underset{D}{(0,5)}$ is a square.

Ans:

$AB=\sqrt{5^2+1^2}=\sqrt{26}$

$BC=\sqrt{1^2+5^2}=\sqrt{26}$

$CD=\sqrt{5^2+1^2}=\sqrt{26}$

$DA=\sqrt{1^2+5^2}=\sqrt{26}$

$AC=\sqrt{(1-5)^2+6^2} \Rightarrow AC=2\sqrt{13}$

$BD=\sqrt{6^2+(1-5)^2} \Rightarrow BD=2\sqrt{13}$

The diagonals

$\because AB=BC=CB=DA=\sqrt{26}$, $AC=BD=2\sqrt{13}$, Then

$ABCD$ is a square. ///

من خصائص المربع

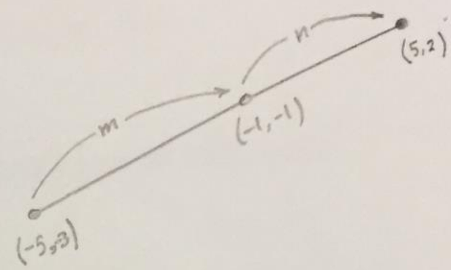
* طول أضلاعه متساوية.

* طول القطرين متساويه.

11. In what ratio does the Point $(-1,-1)$ devide the join of $(-5,-3)$. $(5,2)$?

Ans: for internally devision, we have :

$$-1 = \frac{5m-5n}{m+n} \quad \& \quad -1 = \frac{2m-3n}{m+n}$$



Then

$$5m-5n = 2m-3n \Rightarrow 3m = 2n$$

$$\therefore \boxed{m : n = 2 : 3}$$

12. Find the areas of the triangle of the following Vertices :
i. $(0,0)$. $(12,0)$ and $(0,5)$. ii. $(-2,3)$, $(4,3)$ and $(1,1)$.

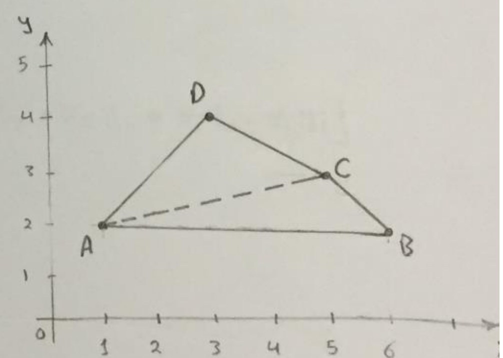
Ans:

The area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, ~~then~~

i. $A = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 12 & 0 & 1 \\ 0 & 5 & 1 \end{vmatrix} = \frac{1}{2}\left[1(60)\right] = 30$ sq. unit

ii. $A = \frac{1}{2}\begin{vmatrix} -2 & 3 & 1 \\ 4 & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left|\left[1+5+6\right]\right| = 6$ sq. unit

13. Find the area of the quadrilateral whose vertices taken order are
$A=(1,2)$. $B=(6,2)$. $C=(5,3)$ and $D=(3,4)$



Ans:

The area of $ABCD$ = area of ΔACD + area of ΔABC

and

area of $\Delta ABC = \frac{1}{2}\left|\begin{vmatrix} 1 & 2 & 1 \\ 6 & 2 & 1 \\ 5 & 3 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|\left[8+7+(-10)\right]\right| = \frac{5}{2}$

,

area of $\Delta ACD = \frac{1}{2}\left|\begin{vmatrix} 1 & 2 & 1 \\ 5 & 3 & 1 \\ 3 & 4 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|\left[11+2+(-7)\right]\right| = 3$

Then, $\boxed{\text{The area of } ABCD = 5.5 \text{ sq. unit}}$

14. for $A=(3,1)$, $B=(7,-3)$, $C=(8,-1)$, $D=(19,-3)$ Prove that area of ΔABC and of ΔADC are equal in magnitude but opposite in sign.

Ans:

$$\text{area of } ABC = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 7 & -3 & 1 \\ 8 & -1 & 1 \end{vmatrix} = \frac{1}{2}\left[-7+24-(-3-8)+(-9-7)\right] = 6 \text{ sq. unit}$$

,

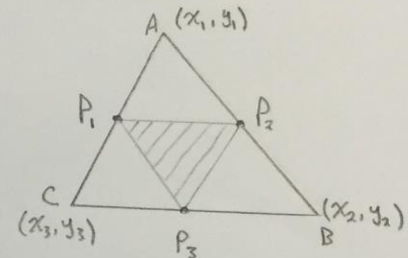
$$\text{area of } ADC = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 19 & -3 & 1 \\ 8 & -1 & 1 \end{vmatrix} = \frac{1}{2}\left[-19+24-(-3-8)+(-9-19)\right] = -6 \text{ sq. unit}$$

Then the area of ΔABC and ΔADC are equal in magnitude and opposite in sign.

15. Find the Coordinates of the middle Points of the sides of the triangle whose Vertices are (x_1, y_1), (x_2, y_2), (x_3, y_3) and Prove that the area of the triangle formed by joining these Points is one-fourth of that of the original triangle.

P4

Ans:



$$P_1 = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right),$$

$$P_2 = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right),$$

$$P_3 = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

,

$$\text{area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\left[x_2 y_3 - x_3 y_2 - x_1 y_3 + x_3 y_1 + x_1 y_2 - x_2 y_1\right]$$

,

$$\text{area of } \Delta P_1 P_2 P_3 = \frac{1}{2}\begin{vmatrix} \frac{x_1+x_3}{2} & \frac{y_1+y_3}{2} & 1 \\ \frac{x_1+x_2}{2} & \frac{y_1+y_2}{2} & 1 \\ \frac{x_2+x_3}{2} & \frac{y_2+y_3}{2} & 1 \end{vmatrix}$$

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$$= \frac{1}{2}\left\{\frac{1}{4}\left[(x_1+x_2)(y_2+y_3) - (x_2+x_3)(y_1+y_2)\right] - \frac{1}{4}\left[(x_1+x_3)(y_2+y_3) - (x_2+x_3)(y_1+y_3)\right] + \frac{1}{4}\left[(x_1+x_3)(y_1+y_2) - (x_1+x_2)(y_1+y_3)\right]\right\}$$

$$= \frac{1}{2}\cdot\frac{1}{4}\left[(x_2-x_3)(y_2+y_3) + (x_1-x_2)(y_1+y_2) + (x_3-x_1)(y_1+y_3)\right]$$ فكها

$$= \frac{1}{4}\cdot\frac{1}{2}\left[x_2 y_3 - x_3 y_2 - x_1 y_3 + x_3 y_1 + x_1 y_2 - x_2 y_1\right], \text{ Then}$$

$$\boxed{\text{area of } \Delta P_1 P_2 P_3 = \frac{1}{4} \text{ area of } \Delta ABC}$$

16. Show that the Points (0,4), (3,2), (6,0) are colinear.

P4

Ans:

$$D = \begin{vmatrix} 0 & 4 & 1 \\ 3 & 2 & 1 \\ 6 & 0 & 1 \end{vmatrix} = (3*0-12)-(0-24)+(0-12) = -12+24-12 = 0$$

$$\because \; D = \begin{vmatrix} 0 & 4 & 1 \\ 3 & 2 & 1 \\ 6 & 0 & 1 \end{vmatrix} = 0$$, Then (0,4), (3,2) & (6,0) are colinear.

17. Find λ for colinear Points $(0,\lambda)$, $(-2,1)$, $(-3,-2)$.

Ans:

$$\begin{vmatrix} 0 & \lambda & 1 \\ -2 & 1 & 1 \\ -3 & -2 & 1 \end{vmatrix} = 0 \Rightarrow 7 \;\; 3\lambda + 2\lambda = 0 \Rightarrow$$

$$4+3-(0+3\lambda)+(0+2\lambda) = 0 \Rightarrow 7-3\lambda+2\lambda = 0 \Rightarrow \boxed{\lambda = 7}$$

Exercise (2) Ch. 1

هندسة اعدادي

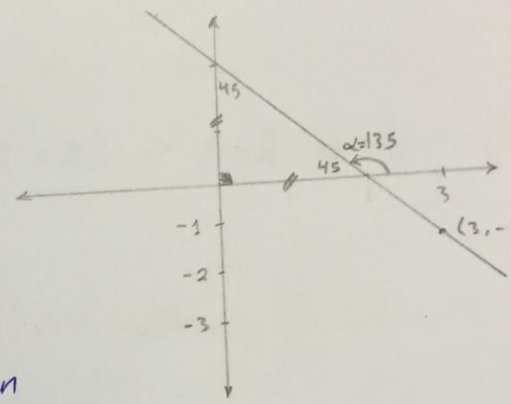
1. Find eq.n of the line of slope -1 and passes through $(4,1)$.

Ans: $y - y_1 = m(x - x_1) \Rightarrow y - 1 = -1(x - 4)$

$\therefore \boxed{y + x - 5 = 0}$ #

2. Find eq.n of the line passes through $(3,-1)$ and makes equal intercepts on the axes.

Ans.



slope $= \tan\alpha = \tan(135) = -1$, Then

$y + \quad = -1(x-3) \Rightarrow \boxed{y + x - 2 = 0}$ #

or The eq.n of line that intersect a from x-axis and b from y-axis is

$\frac{x}{a} + \frac{y}{b} = 1$ but $a = b$ (Given), Then

$\frac{x+y}{a} = 1 \Rightarrow x + y = a$ is the eq.n of the line

sub. with $(3,-1)$ in it, Then $a = 3 - 1 = 2$, hence

$\boxed{y + x - 2 = 0}$ is the eq.n of the line. #

3. Find eq.n of the line that passes through $(-4,2)$ & $(1,-3)$.

Ans: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{y-2}{x+4} = \frac{-3-2}{1+4} \Rightarrow y - 2 = -x - 4$

$\therefore \boxed{y + x + 2 = 0}$ #

4. Find eq.n of the line that intercept 3 on x-axis and 4 on y-axis.

Ans: $\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{3} + \frac{y}{4} = 1 \quad * \frac{12}{12}$

$\therefore 4x + 3y - 12 = 0 \Rightarrow \boxed{3y + 4x - 12 = 0}$ #

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5. Prove that the line through the points (4,3) and (2,5) cuts off equal intercepts on the axes.

Ans. let the eq.n of the line is $\frac{x}{a} + \frac{y}{b} = 1$

we want to prove that $a = b$, then we will sub. with (4,3) and (2,5) to get two eq.ns which we will solve to get a and b, i.e.

$\frac{4}{a} + \frac{3}{b} = 1$ & $\frac{2}{a} + \frac{5}{b} = 1$. Put $A = \frac{1}{a}$ and $B = \frac{1}{b}$

$\therefore$ $4A + 3B = 1 \rightarrow (1)$ & $2A + 5B = 1 \rightarrow (2)$

$(1) = (2) \Rightarrow 4A + 3B = 2A + 5B \Rightarrow 2A = 2B \Rightarrow A = B \Rightarrow \frac{1}{a} = \frac{1}{b}$

$\therefore \boxed{a = b}$

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6. Find the eq.n of the line passes through (3,6) and makes angle $\tan^{-1} 3$ with x-axis.

Ans.

$\alpha = \tan^{-1} 3 \Rightarrow$ slope $= \tan\alpha = 3$.

$y - y_1 = m(x - x_1) \Rightarrow y - 6 = 3(x - 3) \Rightarrow \boxed{y - 3x + 3 = 0}$

7. Find eq.n of line joining (3,2) and (-1,-5).

Ans. $\frac{y-2}{x-3} = \frac{-7}{-4} \Rightarrow 4(y-2) = 7(x-3) \Rightarrow \boxed{4y - 7x + 13 = 0}$

8. Find eq.n of the line through (2,-1) and normal to $4x - 3y = 6$.

Ans.

$-3y + 4x - 6 = 0 \Rightarrow$ has slope $= \frac{-4}{-3} = \frac{4}{3} = m_1$

for perpendicular lines $m_1 m_2 = -1 \Rightarrow \frac{4}{3} * m_2 = -1 \Rightarrow m_2 = \frac{-3}{4}$

$\therefore$ eq.n of the line will be $y + 1 = \frac{-3}{4}(x - 2) \Rightarrow 4y + 4 = -3x + 6$

$\therefore \boxed{4y + 3x - 2 = 0}$

9. Find the bisectors of the angle betn $4x - 3y = 6$ & $2x + 3y = 12$

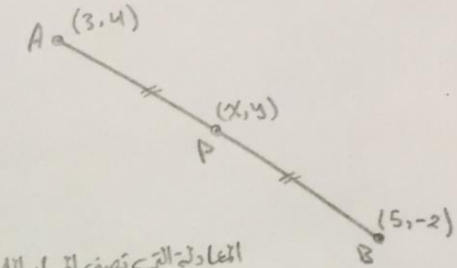
$$\frac{4x - 3y - 6}{\sqrt{16+9}} = \pm \frac{2x + 3y - 12}{\sqrt{4+9}}$$

Ans.

تمرين

Exercise (3) ch.1 هندسة إعدادي

1. A Point P moves so that its distances from two Points (3, 4) and (5, −2) are equal to one another. Find the eq.n of the locus of P.



Ans: $AP = BP$, Then

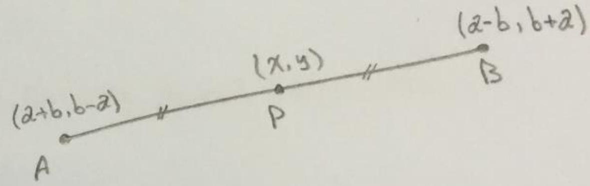
$$(x-3)^2 + (y-4)^2 = (x-5)^2 + (y+2)^2 \text{, Then}$$

$$\cancel{x^2} - 6x + 9 + \cancel{y^2} - 8y + 16 = \cancel{x^2} - 10x + 25 + \cancel{y^2} + 4y + 4$$

$$4x - 12y - 4 = 0 \Rightarrow \boxed{x - 3y - 1 = 0} \rightarrow \text{locus of } P$$

المعادلة التي تصف المسار اللي هتتحرك عليه النقطة P من A إلى B بحيث تكون في منتصف الما

2. Prove that the locus of a Point which is equidistance from the Point $(a+b, b-a)$ and $(a-b, b+a)$ is $bx = ay$.



Ans: $AP = BP$, Then

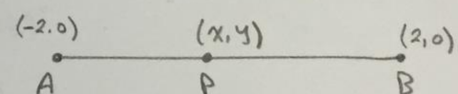
$$(x-(a+b))^2 + (y-(b-a))^2 = (x-(a-b))^2 + (y-(b+a))^2$$

$$-2x(a+b) + \cancel{(a+b)^2} - 2y(b-a) + \cancel{(b-a)^2} = -2x(a-b) + \cancel{(a-b)^2} - 2y(b+a) + \cancel{(b+a)^2}$$

$$2x(\cancel{a}-b-\cancel{a}-b) = 2y(\cancel{b}-a-\cancel{b}-a) \Rightarrow -4bx = -4ay \Rightarrow \boxed{bx = ay}$$

P10 3. The sum of the squares of the distance of a moving Point from the two fixed Points (2, 0) and (−2, 0) is equal to 16. Find the eq.n of it's locus

Ans:



$$(AP)^2 + (BP)^2 = 16$$

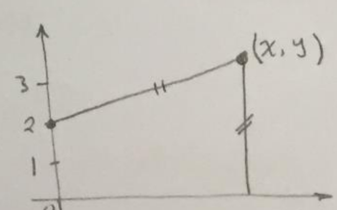
$$(x+2)^2 + (y)^2 + (x-2)^2 + (y)^2 = 16$$

$$2x^2 + 2y^2 + 8 = 16 \Rightarrow \boxed{x^2 + y^2 = 4}$$

P10 4. A Point P moves so that its distance from the fixed Point (0, 2) is equal to its distance from the x-axis. Prove that the eq.n of the locus is $x^2 = 4(y-1)$

Ans:

بعد النقطة (x, y) عن محور x يساوي الـ y



$$\sqrt{(x-0)^2 + (y-2)^2} = y$$

$$x^2 + \cancel{y^2} - 4y + 4 = \cancel{y^2} \Rightarrow \boxed{x^2 = 4(y-1)} \;\divideontimes$$

↳ Parabola

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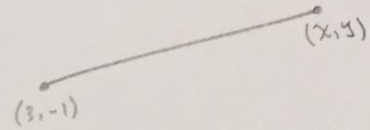
5. Find the eq.n of the locus of a Point which is at a distance 3 from the Point (3,-1).

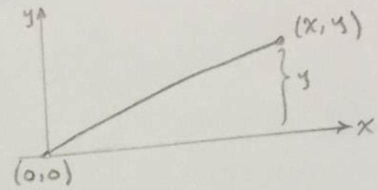


Ans:

$\sqrt{(x-3)^2+(y+1)^2} = 3 \Rightarrow x^2-6x+9+y^2+2y+1=9$

$\therefore \boxed{x^2+y^2-6x+2y+1=0}$

6. A Point moves so that its distance from x-axis is half of its distance from the origin. Find the eq.n of its locus.



Ans:

$y = \frac{1}{2}\sqrt{x^2+y^2} \Rightarrow x^2+y^2-2y^2=0$

$\therefore \boxed{x^2-y^2=0}$

7. Find the eq.n of the locus of a Point whose distance from (-1,1) is equal to twice its distance from the origin.

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Ans:

$\sqrt{(x+1)^2+(y-1)^2} = 2\sqrt{(x-0)^2+(y-0)^2} \Rightarrow x^2+2x+1+y^2-2y+1 = 4x^2+4y^2$

Then

$\boxed{3x^2-2x+3y^2+2y-2=0}$

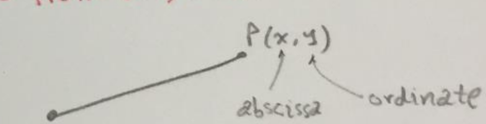
P11 8. Given A (a,b), B (3a,3b) show that if P(x,y) is a Point such that PA = PB Then $ax+by=2(a^2+b^2)$.

Ans: $PA = PB \Rightarrow (x-a)^2+(y-b)^2 = (x-3a)^2+(y-3b)^2$

Then $-2ax+a^2-2by+b^2 = -6ax+9a^2-6by+9b^2$

$4ax+4by = 8a^2+8b^2 \Rightarrow \boxed{ax+by=2(a^2+b^2)}$

P11 9. Find locus of P which moves such that its distance from (0,3) is equal to the ordinate of P.



Ans:

$\sqrt{x^2+(y-3)^2} = y \Rightarrow x^2+\cancel{y^2}-6y+9 = \cancel{y^2}$

$\boxed{x^2-6y+9=0}$

P11 10. Consider P(x,y) moves such that the difference bet.n PA & PB equal 8 where A (-5,0), B(5,0). Find locus of P. --> [ellipse eq n]

Ans:

$\sqrt{(x+5)^2+y^2} - \sqrt{(x-5)^2+y^2} = 8 \rightarrow \sqrt{x^2+10x+25+y^2} = 8+\sqrt{x^2-10x+25+y^2}$ by squaring

Then make abbriviation u get $x-16 = 4\sqrt{x^2-10x+25+y^2}$, by squaring again

$\boxed{9x^2+80x-16y^2-144=0}$

Exercise (4) ch.1 هندسة إعدادي

1. Find the distances betⁿ:

(i) $(3, 60^\circ)$ and $(5, 150^\circ)$. (ii) $(6, 30)$ and $(4, 90)$

(iii) $(2, 40)$ and $(4, 100)$. (iv) $(2a, 30)$ // $(4a, 120)$

Ans. for two points $P = (r_1, \theta_1)$ and $Q = (r_2, \theta_2)$. the length of $\overline{PQ}$ is

$PQ = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_1 - \theta_2)}$, Then

(i) $PQ = \sqrt{9 + 25 - 30 \cos(90)} \Rightarrow \boxed{PQ = \sqrt{34}}$ #

(ii) $PQ = \sqrt{36 + 16 - 48 \cos(60)} \Rightarrow \boxed{PQ = 2\sqrt{7}}$ #

(iii) $PQ = \sqrt{4 + 16 - 16 \cos(60)} \Rightarrow \boxed{PQ = 2\sqrt{3}}$ #

(iv) $PQ = \sqrt{4a^2 + 16a^2 - 16a \cos(90)} \Rightarrow \boxed{PQ = 2\sqrt{5}\, a}$ #

2. Prove that the points $(0, 0)$, $(3, 90^\circ)$ and $(3, 30^\circ)$ form and equilateral triangle.

A, B, C

Ans.

$AB = \sqrt{0^2 + 3^2 - 2 * 0 * 3 \cos(90)} \Rightarrow AB = 3$

, $BC = \sqrt{3^2 + 3^2 - 2 * 3 * 3 \cos(60)} \Rightarrow BC = 3$

, $CA = \sqrt{0^2 + 3^2 - 2 * 0 * 3 \cos(30)} \Rightarrow CA = 3$

$\because AB = BC = CA$. Then A, B and C are vertices of equilateral triangle.

3. Find the areas of the triangles with vertices:

i. $(2, 60^\circ)$, $(3, 90^\circ)$ and $(4, 120^\circ)$.

ii. $(-6, 30^\circ)$, $(4, 90^\circ)$ // $(5, 150^\circ)$. $(6, 210)$, $(4, 90)$ & $(5, 150)$

iii. $(2a, 30^\circ)$, $(4a, 60)$ // $(6a, 90)$.

Ans. The area of a triangle $ABC = \frac{1}{2}\left[r_1 r_2 \sin(\theta_2 - \theta_1) + r_2 r_3 \sin(\theta_3 - \theta_2) + r_3 r_1 \sin(\theta_1 - \theta_3)\right]$

i. $A = \frac{1}{2}\left[2 \times 3 \sin(30) + 3 * 4 \sin(30) + 4 * 2 \sin(-60)\right] \Rightarrow \boxed{A = 1.036 \text{ sq. unit}}$ #

ii. $A = \frac{1}{2}\left[6 * 4 \sin(-120) + 4 * 5 \sin(60) + 5 * 6 \sin(60)\right] \Rightarrow \boxed{A = 11.26 \text{ sq. unit}}$ #

iii. $A = \frac{1}{2}\left[2a * 4a \sin(30) + 4a * 6a \sin(30) + 6a * 2a \sin(-60)\right] \Rightarrow \boxed{A = 2.804\, a^2 \text{ sq. unit}}$ #

4. Transform into the Corresponding Polar Coordinates:- Put $x = r\cos\theta$, $y = r\sin\theta$

i. $3x + y = 0$

Ans.

$3r\cos\theta + r\sin\theta = 0$

$\therefore \boxed{3\cos\theta + \sin\theta = 0} \Rightarrow \sin\theta = -3\cos\theta \Rightarrow \tan\theta = -3 \Rightarrow \boxed{\theta = 288.43}$

ii. $x^2 + y^2 = 16$

Ans.

$r^2\cos^2\theta + r^2\sin^2\theta = 16 \Rightarrow r^2\left[\cancel{\cos^2\theta + \sin^2\theta}\right]^{1} = 16 \Rightarrow \boxed{r^2 = 16} \Rightarrow \boxed{r = 4}$

iii. $y^2 = 4ax$

Ans. $r^{\cancel{2}}\sin^2\theta = 4a\cancel{r}\cos\theta \Rightarrow \boxed{r\sin^2\theta = 4a\cos\theta}$

iv. $(x^2+y^2)^2 = 2a^2xy$

Ans. $(r^2\cos^2\theta + r^2\sin^2\theta)^2 = 2a^2 \cdot r^2\sin\theta\cdot\cos\theta \Rightarrow r^4 = a^2r^2\sin(2\theta)$

$\therefore \boxed{r^2 = a^2\sin 2\theta}$

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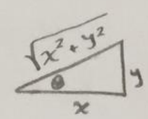
V. $x^2 + y^2 - 2x + 2y = 0$

Ans. $\underbrace{r^2\cos^2\theta + r^2\sin^2\theta}_{r^2} - 2r\cos\theta + 2r\sin\theta = 0$

$r^2 - 2\cancel{r}(\cos\theta + \sin\theta) = 0 \Rightarrow \boxed{r = 2(\cos\theta + \sin\theta)}$

5. Transform into the Corresponding Cartesian Coordinates: Put $r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$



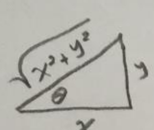
i. $r^2 = a^2\cos 2\theta \Rightarrow r^2 = a^2[\cos^2\theta - \sin^2\theta]$

Ans.

$x^2 + y^2 = a^2\left[\frac{x^2}{x^2+y^2} - \frac{y^2}{x^2+y^2}\right] \Rightarrow \boxed{(x^2+y^2)^2 = a^2(x^2 - y^2)}$ ##

ii. $r = 8\cos\theta$

Ans. Put $r = \sqrt{x^2+y^2}$, $\tan\theta = \frac{y}{x}$



$\Rightarrow \cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

$\therefore r = 8\cos\theta \Rightarrow \sqrt{x^2+y^2} = \frac{8x}{\sqrt{x^2+y^2}} \Rightarrow \boxed{x^2 - 8x + y^2 = 0}$ ##

iii. $r = \frac{4}{1+\cos\theta}$ (استخدم تعويض المسألة السابقة)

Ans. $\sqrt{x^2+y^2} = \frac{4}{1 + \frac{x}{\sqrt{x^2+y^2}}} \Rightarrow \cancel{\sqrt{x^2+y^2}} = \frac{4\cancel{\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2} + x} \Rightarrow \boxed{\sqrt{x^2+y^2} - x = 4}$ ##

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Exercise (5) Ch.1 هندسة - إعدادي

1. Change to a new origin in each case:

i. $x^2+y^2+2x-4y+1=0$; new origin $(-1,2)$.

ii. $x^2+2y^2-6x+16y+39=0$; // // $(3,-4)$.

Ans. Put $x=x'+h$ and $y=y'+k$ for new origin (h,k).

i. $x^2+y^2+2x-4y+1=0$, Put $x=x'-1$, $y=y'+2$, Then

$(x'-1)^2+(y'+2)^2+2(x'-1)-4(y'+2)+1=0$

$x'^2-\cancel{2x'}+\cancel{1}+y'^2+\cancel{4y'}+4+\cancel{2x'}-2-\cancel{4y'}-8+\cancel{1}=0$

Then: $\boxed{x'^2+y'^2-4=0}$

ii. $x^2+2y^2-6x+16y+39=0$ Put $x=x'+3$, $y=y'-4$, then

$(x'+3)^2+2(y'-4)^2-6(x'+3)+16(y'-4)+39=0$;

$x'^2+\cancel{6x'}+9+2y'^2-\cancel{16y'}+32-\cancel{6x'}-18+\cancel{16y'}-64+39=0$

Then: $\boxed{x'^2+2y'^2-2=0}$

2. write down the eq.ns for a rotation of axes by $\frac{\pi}{4}$, hence prove that the curve $2xy=a^2$ can be transformed to $x^2-y^2=a^2$.

Ans. $x=x'\cos\theta-y'\sin\theta$, $y=x'\sin\theta+y'\cos\theta$ $\Big\}$ $x=\frac{x'}{\sqrt{2}}-\frac{y'}{\sqrt{2}} \Rightarrow \boxed{\sqrt{2}\,x=x'-y'}$, $y=\frac{x'}{\sqrt{2}}+\frac{y'}{\sqrt{2}} \Rightarrow \boxed{\sqrt{2}\,y=x'+y'}$ The req. eq.ns $\rightarrow$ (1)

sub. with (1) in $2xy=a^2$ where,

$\sqrt{2}\,x\cdot\sqrt{2}\,y=a^2 \Rightarrow (x'-y')\cdot(x'+y')=a^2 \Rightarrow \boxed{x'^2-y'^2=a^2}$

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3. Transform $x^2-6xy+9y^2+4x+8y+15=0$ to new axes through $(-2,1)$ rotated by 45°.

$(h = -2,\ K = 1,\ \theta = 45^\circ)$

Ans. Put $x = h + x'\cos\theta - y'\sin\theta \Rightarrow x = -2 + \frac{x'}{\sqrt2} - \frac{y'}{\sqrt2} = -2 + \frac{x'-y'}{\sqrt2}$

, $y = K + x'\sin\theta + y'\cos\theta \Rightarrow y = 1 + \frac{x'}{\sqrt2} + \frac{y'}{\sqrt2} = 1 + \frac{x'+y'}{\sqrt2}$

sub. in the given eq.n , Then

$$\left(\frac{x'-y'}{\sqrt2}-2\right)^2 - 6\left(\frac{x'-y'}{\sqrt2}-2\right)\left(\frac{x'+y'}{\sqrt2}+1\right) + 9\left(\frac{x'+y'}{\sqrt2}+1\right)^2 + 4\left(\frac{x'-y'}{\sqrt2}-2\right) + 8\left(\frac{x'+y'}{\sqrt2}+1\right) + 15 = 0 \rightarrow (1)$$

$$\left(\frac{x'-y'}{\sqrt2}-2\right)^2 = \frac{(x'-y')^2}{2} - \frac{4}{\sqrt2}(x'-y') + 4 = \frac{1}{2}(x'^2-2x'y'+y'^2) - \frac{4}{\sqrt2}x' + \frac{4}{\sqrt2}y' + 4$$

$$\left(\frac{x'+y'}{\sqrt2}+1\right)^2 = \frac{(x'+y')^2}{2} + \frac{2}{\sqrt2}(x'+y') + 1 = \left(\frac{1}{2}(x'^2+2x'y'+y'^2) + \frac{2}{\sqrt2}x' + \frac{2}{\sqrt2}y' + 1\right)$$ ← خدبالك ده مضروب في 9

$$\left(\frac{x'-y'}{\sqrt2}-2\right)\left(\frac{x'+y'}{\sqrt2}+1\right) = \left(\frac{x'^2-y'^2}{2} + \frac{x'-y'}{\sqrt2} - \frac{2}{\sqrt2}(x'+y') - 2\right)$$ ← خدبالك ده مضروب في (-6)

Then the eq.n (1) will be:

$$x'^2\left(\frac{1}{2}+\frac{9}{2}-\frac{6}{2}\right) + y'^2\left(\frac{1}{2}+\frac{9}{2}+\frac{6}{2}\right) + x'y'(-1+9) + x'\left(\frac{4}{\sqrt2}+\frac{8}{\sqrt2}-\frac{4}{\sqrt2}+\frac{18}{\sqrt2}-\frac{6}{\sqrt2}+\frac{12}{\sqrt2}\right)$$

$$+ y'\left(\frac{4}{\sqrt2}+\frac{6}{\sqrt2}+\frac{12}{\sqrt2}+\frac{18}{\sqrt2}-\frac{4}{\sqrt2}+\frac{8}{\sqrt2}\right) + (4+12+9+8-8+15) = 0$$

$\therefore\ 2x'^2 + 8y'^2 + 16\sqrt2\,x' + 22\sqrt2\,y' + 40 + 8x'y' = 0$

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or $\boxed{x'^2 + 4y'^2 + 4x'y' + 8\sqrt2\,x' + 11\sqrt2\,y' + 20 = 0}$ #

4. Transform $3x^2-24xy+10y^2+6x+52y=0$ to new origin $(3,1)$ and rotated by 45.

Ans. Put $x = h + x'\cos\theta - y'\sin\theta \Rightarrow x = 3 + \frac{x'-y'}{\sqrt2} \Rightarrow \boxed{x = \frac{x'-y'}{\sqrt2}+3}$

$y = K + x'\sin\theta + y'\cos\theta \Rightarrow y = 1 + \frac{x'+y'}{\sqrt2} \Rightarrow \boxed{y = \frac{x'+y'}{\sqrt2}+1}$

$$3x^2 = 3\left(\frac{x'-y'}{\sqrt2}+3\right)^2 = \frac{3}{2}(x'^2-2x'y'+y'^2) + \frac{18}{\sqrt2}(x'-y') + 27$$

,

$$-24xy = -24\left(\frac{x'-y'}{\sqrt2}+3\right)\left(\frac{x'+y'}{\sqrt2}+1\right) = -12(x'^2-y'^2) - \frac{24}{\sqrt2}(x'-y') - \frac{72}{\sqrt2}(x'+y') - 72$$

,

$$10y^2 = 10\left(\frac{x'+y'}{\sqrt2}+1\right)^2 = 5(x'^2+2x'y'+y'^2) + \frac{20}{\sqrt2}(x'+y') + 10$$

,

$$6x = \frac{6}{\sqrt2}(x'-y') + 18$$

,

$$52y = \frac{52}{\sqrt2}(x'+y') + 52$$, Then the given eq.n will be:

$$x'^2\left(\frac{3}{2}-12+5\right) + y'^2\left(\frac{3}{2}+5+12\right) + x'y'(-3+10) + x'\left(\frac{18}{\sqrt2}-\frac{24+72}{\sqrt2}+\frac{20}{\sqrt2}+\frac{6+52}{\sqrt2}\right) \rightarrow 0$$

$$+ \frac{y'}{\sqrt2}(-18+24-72+20-6+52) \rightarrow 0 + (27-72+10+18+52) = 0 ,\quad \text{or}$$

$$-\frac{11}{2}x'^2 + \frac{37}{2}y'^2 + 7x'y' + 35 = 0 \Rightarrow \boxed{11x'^2 - 37y'^2 - 14x'y' - 70 = 0}$$ #

باقي مسائل هذا السؤال
تمانفس الفكرة

6. Find (h, k) for no 1st degree terms for

iv. $2x^2 + 2y^2 - 8x + 5 = 0$

Ans. Put $x = x' + h$, $y = y' + k$, Then

$2x^2 + 2y^2 - 8x + 5 = 0$,

$2(x'+h)^2 + 2(y'+k)^2 - 8(x'+h) + 5 = 0$, Then

$2x'^2 + 4x'h + 2h^2 + 2y'^2 + 4ky' + 2k^2 - 8x' - 8h + 5 = 0$

or

$$2x'^2 + 2y'^2 + (4h-8)x' + (4k)y' + 2h^2 + 2k^2 - 8h + 5 = 0 \longrightarrow (1)$$

for no 1st degree terms put

$4h - 8 = 0 \Rightarrow h = 2$ and $4k = 0 \Rightarrow k = 0$; Then

$\boxed{(2, 0)}$ is the Point of new origin . Then the new eq.1 will be sub. with $(2,0)$ in (1).

$$\boxed{2x'^2 + 2y'^2 - 3 = 0}$$

8. Find angle of rotation for no $x'y'$ term:

iii. $\underbrace{x^2}_{A} \underbrace{- 3xy}_{B} + \underbrace{4y^2}_{C} + 7 = 0$

Ans.

$$2\alpha = \tan^{-1}\left(\frac{B}{A-C}\right) \Rightarrow 2\alpha = \tan^{-1}\left(\frac{-3}{1-4}\right) \Rightarrow 2\alpha = 45 \Rightarrow \boxed{\alpha = 22.5}$$

i. $3xy + y - 2 = 0$

Ans.

$$2\alpha = \tan^{-1}\left(\frac{B}{A-C}\right) = \tan^{-1}\left(\frac{3}{0-0}\right) \Rightarrow 2\alpha = 90 \Rightarrow \boxed{\alpha = 45}$$

المسائل (5) و(7) تطبيق مباشر القوانين

وشكراً

(^^) , محمد عماد 3/3 2016 (^^)

by / Mohammed Emad